

## Feedback Control of Bipedal Locomotion



University of Michigan

**Jessy W. Grizzle**

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## Acknowledgements



**Hae Won Park**  
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Dept. of Mechanical Engineering  
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**RABBIT**  
ROBEA Project  
France

## Videos and Papers

- <http://www.youtube.com/user/DynamicLegLocomotion>
- <http://web.eecs.umich.edu/~grizzle/papers/robotics.html>

## 2011 RAS Pioneer Award to Mark Spong

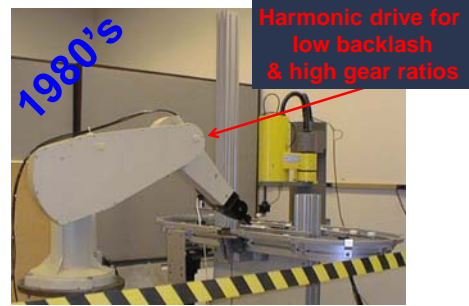
To recognize individuals who, by virtue of initiating new areas of research, [...] have had a significant impact on development of the robotics and/or automation fields.

- Joint elasticity in robot manipulators
- Bilateral teleoperation
- Normal forms for underactuated mechanical systems
- Bipedal locomotion (Passivity and energy shaping, controlled symmetries)

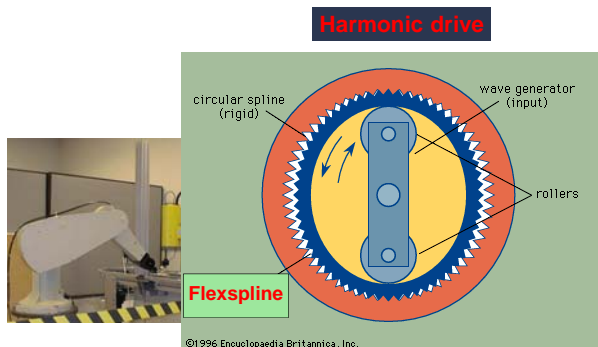
## Outline

- Springs in Robots
  - Mark's problem and approach
  - Jessy's problem
    - approach 1
    - approach 2
    - application to MABEL
- Introduction of the ATRIAS Series of Robots and MARLO (time permitting)

## Manipulators: Designed to be Rigid



## Manipulators with Flexible Joints



## Manipulators with Flexible Joints

- Poor tracking?
- Vibrations in closed loop?

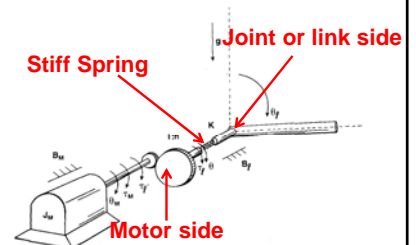


Fig. 1. Single-link manipulator with joint flexibility. (Courtesy M. Spong)

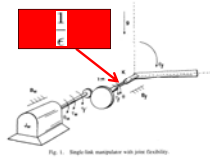
## Manipulators with Flexible Joints

$$D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) + K(q_1 - q_2) = 0$$

$$J\ddot{q}_2 - K(q_1 - q_2) - u = 0.$$

$$K = \frac{1}{\epsilon} I_{n \times n}$$

$$z = \frac{1}{\epsilon}(q_1 - q_2)$$



## Manipulators with Flexible Joints

$$\ddot{q}_1 = -A(q_1)z - H(q_1, \dot{q}_1)$$

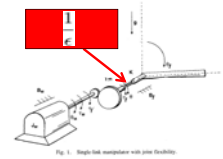
$$\epsilon \ddot{z} = -(A(q_1)z + B)z - H(q_1, \dot{q}_1) - Bu.$$

Composite Control

$$u = u_s(q_1, \dot{q}_1) + u_f(z, \dot{z})$$

“Rigid Model”

“Corrective Term”



## Manipulators with Flexible Joints

$$\ddot{q}_1 = -A(q_1)z - H(q_1, \dot{q}_1)$$

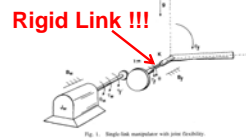
$$\epsilon \ddot{z} = -(A(q_1)z + B)z - H(q_1, \dot{q}_1) - Bu.$$

Composite Control

$$u = u_s(q_1, \dot{q}_1) + u_f(z, \dot{z})$$

“Rigid Model”

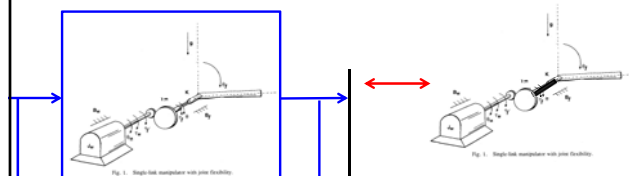
“Corrective Term”



## Manipulators with Flexible Joints

$$u_s + u_f$$

$M$



## Key Papers

- Spong, Khorasani, Kokotovic, "A Slow Manifold Approach to Feedback Control of Nonlinear Flexible Systems," *ACC*, 1985
- **(718 citations)** MW Spong, "Modeling and control of elastic joint robots," *Journal of Dynamic Systems, Measurement, and Control*, 1987. **Implemented worldwide**
- Spong, Khorasani, Kokotovic, "An integral manifold approach to the feedback control of flexible joint robots," *IEEE Trans. Rob. and Automation*, 1987 ,

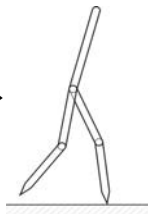
## Bipedal Robots and Springs

## Natural Progression

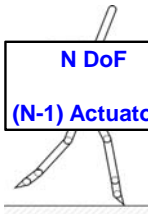
Grizzle, Abba & Plestan **1999**



Plestan, Grizzle, Abba & Westervelt 2000

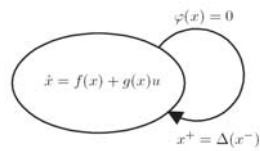


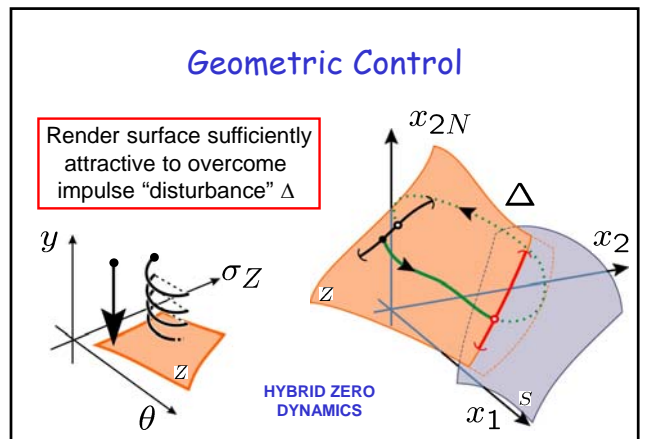
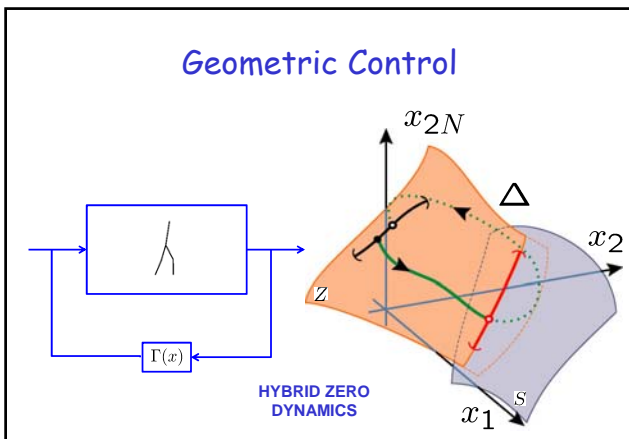
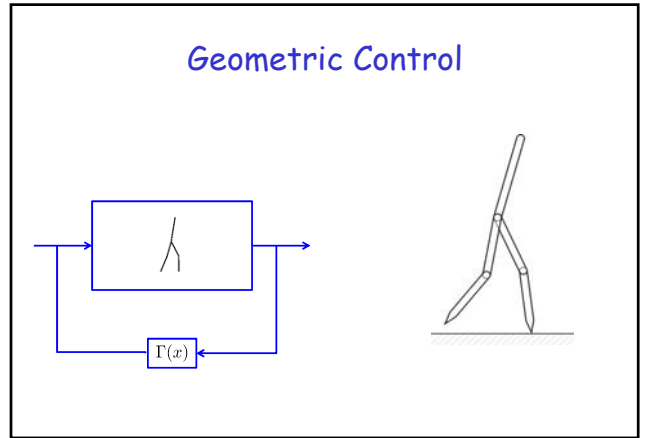
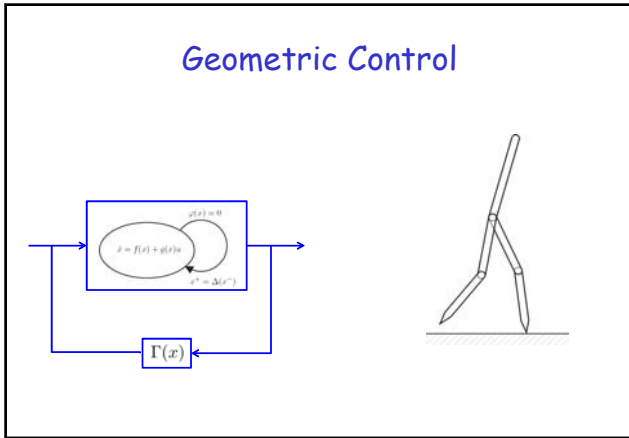
Westervelt, Grizzle & Koditschek **2001**



**N DoF**  
**(N-1) Actuators**

## Geometric Control





## Natural Progression

Grizzle, Abba & Plestan 1999

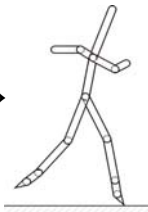


Rabbit



Experiments 2002-2004

Westervelt, Grizzle & Koditschek 2001



## Natural Progression

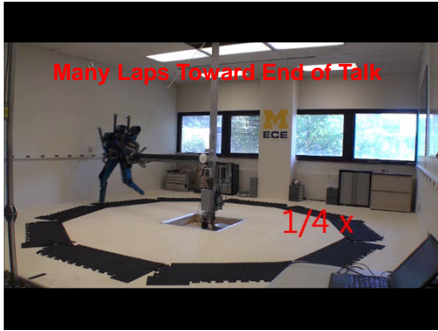
- **Successful walking experiments**
  - Model → controller design → simulation → experiment, with **minimal trial and error**.
  - Various speeds
  - Gait transitions (continuous & discrete)
  - Robustness to unknown loads
  - Robustness to shoves

## 2004 with Rabbit



7 Years of hard labor later ...

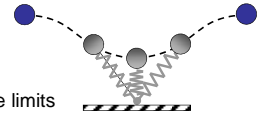
## 7 Years of Work to Get Here



## Why Did Rabbit Not Run Well?

- **Actuator saturation**

- Workspace limitations
- Lossy powertrain
- Nominal gait at 95% of torque limits



- **Motors were asked to emulate a compliant (spring-like), high-bandwidth restoring force**

- Negative work
- Bandwidth issues at impact
- Ground reaction forces

## Teamed up with CMU to Build a Robot with Compliance

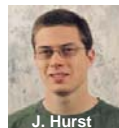


Springs

Torsional Compliance

Planning started in late 2004

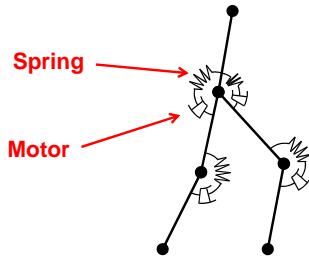
Michigan-CMU Robot



J. Hurst

Springs, Take 1

## Started Analyzing Models + Series Elastic Actuators



## Nominal Robot Without Compliance

**N DoF & (N-1) Actuators**

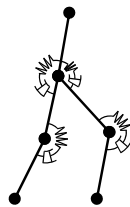


$$q = \begin{bmatrix} q_a \\ \theta \end{bmatrix} = \begin{cases} \text{actuated} \\ \text{body} \end{cases}$$

$$D(q_a)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu,$$

## Robot + SEA

**(2N-1) DoF & (N-1) Actuators**



**Highly Underactuated**

$$\begin{aligned} D(q_a)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) - BK(q_m - q_a) &= 0 \\ \underline{J\ddot{q}_m + K(q_m - q_a)} &= u_c. \end{aligned}$$

## Robot Without SEA

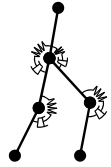
**Feedback Equivalent (Spong, 1994)**



$$\begin{aligned} \dot{\sigma} &= -\frac{\partial V}{\partial \theta}(q) \\ \dot{\theta} &= \frac{\sigma}{d_{NN}(q_a)} + R(q_a)\dot{q}_a \\ \underline{\ddot{q}_a} &= \underline{w}, \end{aligned}$$



## Robot + SEA



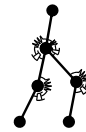
Feedback Equivalent (Isidori + Spong)

$$\begin{aligned}\dot{\sigma} &= -\frac{\partial V}{\partial \theta}(q) \\ \dot{\theta} &= \frac{\sigma}{d_{NN}(q_a)} + R(q_a)\dot{q}_a \\ \underline{q_a^{(4)}} &= w\end{aligned}$$

## Consequences



$$y = h(q_a, \theta)$$



Vector Rel. Degree 2  $\Leftrightarrow$  Vector Rel. Degree 4

Identical Zero Dynamics

(Morris + JG TAC 2009)

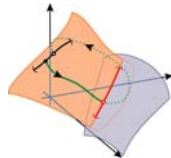
## Consequences



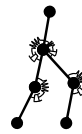
$$y = h(q_a, \theta)$$



Learned how to create invariant manifolds in hybrid models with many degrees of underactuation (TAC 2009)



## Consequences



Design periodic orbit taking advantage of springs

$$y = h(q_a, \theta)$$



### Consequences

$y = h(q_a, \theta)$

Stability analysis  
for a 1 DOF hybrid model

(TAC 2009)

### Springs, Take 2

Ioannis Poulakakis

TAC 2009  
IROS 2007, ICRA 2008

### Modeling Hierarchy

CoM

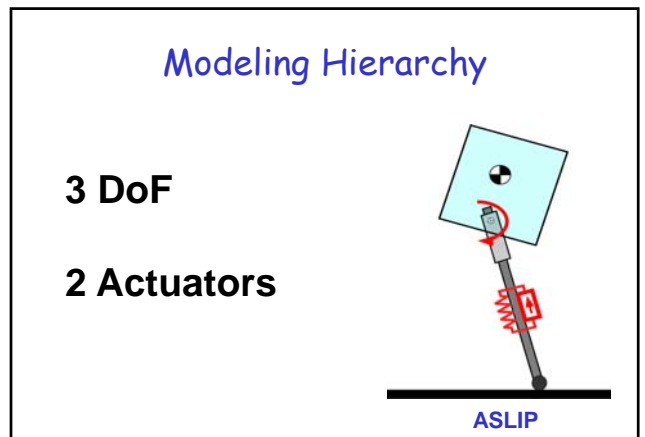
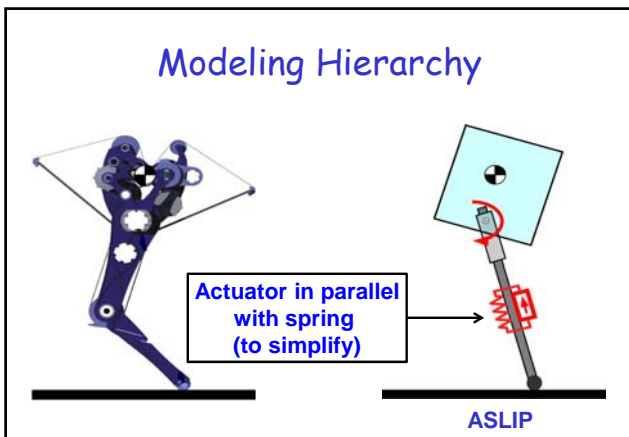
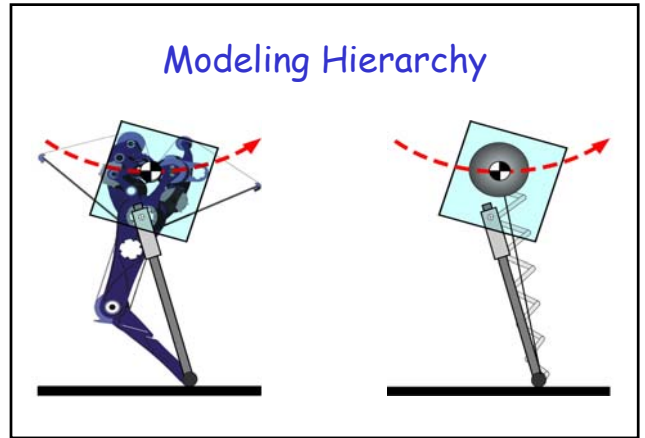
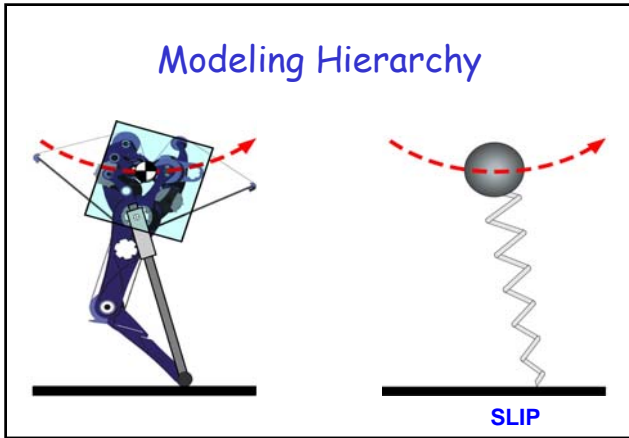
Hip

Torsional Compliance

Michigan-CMU Robot

### Modeling Hierarchy

SLIP=Spring Loaded Inverted Pendulum



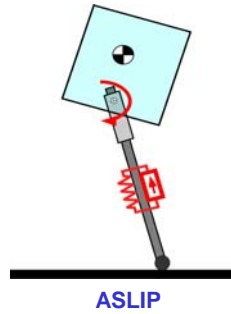
## Modeling Hierarchy

### Key Differences with Literature

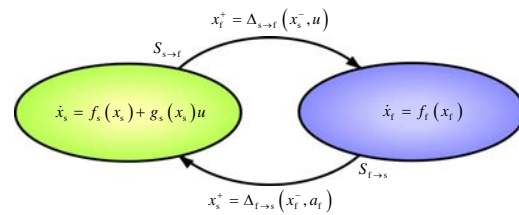
Massive Torso

Hip offset from CoM

Could not be handled by  
Raibert, Koditschek,  
Beuhler, Francois, Samson ...



## ASLIP Hybrid Dynamics: Stance + Flight



## Formal Embedding of the SLIP into an ASLIP Model

**Theorem [IROs'07, TAC 2009]** There exist:

1. A **continuous** feedback controller  $\Gamma_c$  active in the stance of the ASLIP, and an invariant surface  $Z$  (embedded) in the stance state space, such that

$$f_s(x_s) + g_s(x_s)\Gamma_c(x_s)|_Z = \text{SLIP stance phase model}$$

$Z$  is exponentially attractive

2. A **discrete** feedback controller  $\Gamma_f$  active in transitions from flight to stance, such that

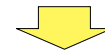
$$\Delta(x_s^-, \Gamma_c(x_s^-), \Gamma_f(x_s^-))|_Z = \text{SLIP reset map}$$

$S_{s \rightarrow f} \cap Z$  is hybrid invariant

## Formal Embedding of the SLIP into an ASLIP Model

Moreover,

If a controller is designed to render a particular orbit of the SLIP exponentially stable the **same** controller will create an exponentially stable orbit in the ASLIP closed-loop system!

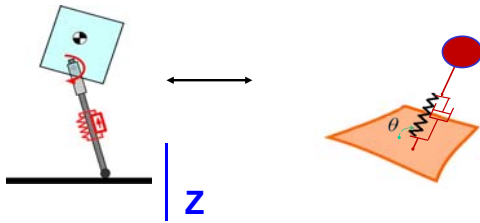


**Controller results available for the SLIP can be directly used in the ASLIP!**

**Caveat:** Embedding is local due to unilateral constraints

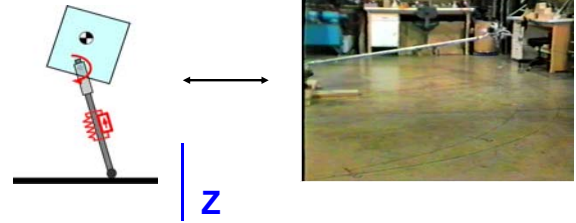
## Theorem in a Picture

There exists a hybrid zero dynamics such that



## Theorem in a Picture

There exists a hybrid zero dynamics such that

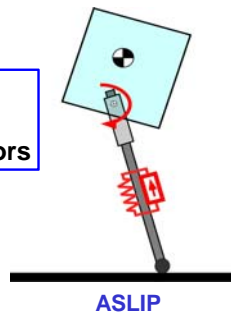


## ASLIP

Can be controlled via two approaches:

- 1 DOF HZD
- 2 DOF HZD

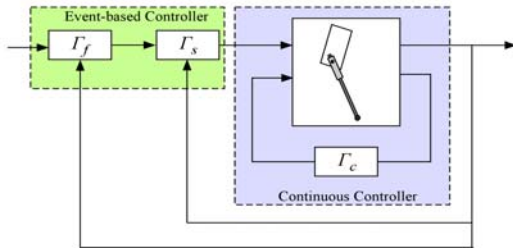
3 DoF  
2 Actuators



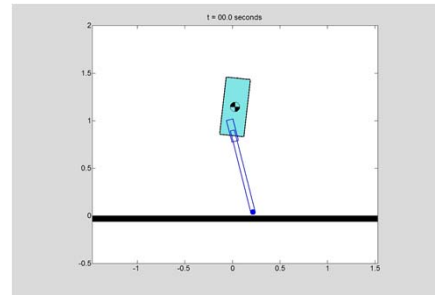
## Comparison of the Two Control Approaches

## Identical Hybrid Controller Structure

- The SLIP embedding and the 1-DOF rigid HZD controller have the same structure, same stability proofs, though different objectives.

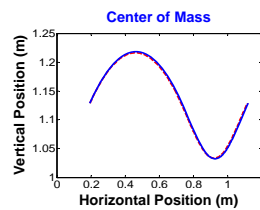


## Identical Periodic Orbit



## Comparison: Steady State

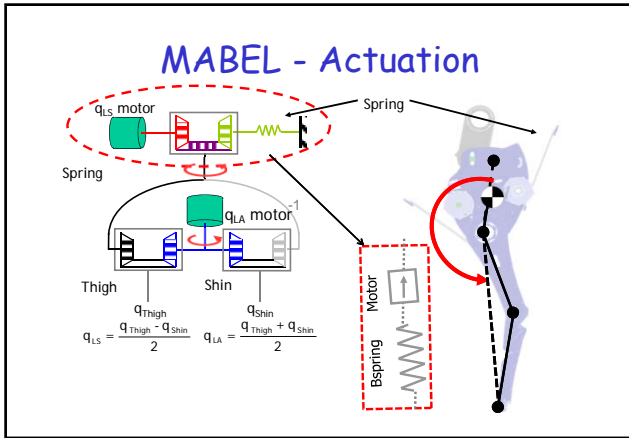
- 1-DOF HZD controller
- SLIP embedding controller
- Same leg actuator force on the nominal orbit. (<0.5% difference)



## Comparison: Transients


Perturbation	Control	Stride	$(u_1^*, u_2^*)^{\max}$	$(W_1, W_2)^{\text{total}}$
$\delta\theta = +3^\circ$	1 DOF	4	(447,15)	(60,18)
	SLIP	3	(36,16)	(18,18)
$\delta\theta = -4^\circ$	1 DOF	13	(493,13)	(125,53)
	SLIP	4	(69,16)	(21,20)
$\delta\dot{x}_c = +0.9\%_s$	1 DOF	12	(448,21)	(241,76)
	SLIP	6	(418,16)	(110,40)
$\delta\dot{x}_c = -0.4\%_s^2$	1 DOF	3	(73,13)	(40,10)
	SLIP	3	(53,23)	(29,13)


Compliant HZD → Larger Domain of Attraction, Less Work by Actuator



### Long Path to Running on MABEL

- Series Elastic Actuation
- Mass in Legs
- Cable Stretch
- Springs too Stiff
- Rotor Inertias + Torque Limits, ...





Koushil Sreenath

### Long Path to Running on MABEL

Koushil Sreenath, Hae-Won Park, Ioannis Poulakakis, and Jessy W. Grizzle, A Compliant Hybrid Zero Dynamics Controller for Stable, Efficient and Fast Bipedal Walking on MABEL, *Int. J. Robotics Research (IJRR)*, 30(9):1170-1193, August 2011.

Koushil Sreenath, Hae-Won Park, and Jessy W. Grizzle, Embedding Active Force Control within the Compliant Hybrid Zero Dynamics to Achieve Stable, Fast Running on MABEL, *in review*.



## ATRIAS Robots & MARLO



## The End

